Fractality of complex networks emerging from self-organized critical dynamics

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Structures of real-world networks are classified into two types from a viewpoint of the relation between the number of nodes and the shortest path length, namely small-world structures [1] where the average path length $\langle l \rangle$ increases at most logarithmically with the network size N and fractal structures [2] where $\langle l \rangle$ scales with N in a power-law manner. Although the small-world property arises from the existence of short-cut edges, the origin of fractality in complex networks still remains unclear. It is thus also not understood why there exist small-world and fractal networks in the real world.

We propose a model of self-organized critical (SOC) dynamics of complex networks and present a possible explanation of the emergence of fractal and small-world networks. A network in our model experiences a continued growth and its occasional decay due to the instability of large grown networks against cascading overload failures. In this work, the description of cascading overload failures is based on the model proposed by Ref. [3] in which temporally fluctuating loads are described by random walkers. We show that cascading failures occur intermittently and prevent networks from growing infinitely. The distribution of the time interval between successive cascades obeys a power-law form. Power-law distributions are also found in both the avalanche size that is the number of eliminated nodes in a single cascade and the cluster size defined as the number of nodes in a connected component. These facts indicate that the network dynamics possesses SOC characteristics. In our model, the load reduction parameter r is set to vary with the network size during the SOC dynamics. When r of the network coincides with its critical value r_c, a cascade of overload failures (critical cascade) decays the network into a critical one. Our result shows that giant components just after critical cascades have fractal structures. The fractal dimension d_B is close to that for the giant component after a critical cascade starting with an Erdős-Rényi random graph. In contrast, networks far from criticality display the small-world property. In particular, we demonstrate the crossover behavior from fractal to small-world structure in a growing process from a critical network, which is caused by short-cut edges introduced by newly added nodes. We have also discussed suitable parameter values to realize SOC dynamics.

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